

Characteristics of a Longitudinal/Transverse Coupling Slot in Crossed Rectangular Waveguides

SEMBIAM R. RENGARAJAN, SENIOR MEMBER, IEEE

Abstract—A rigorous analysis of a broad wall slot coupler between two crossed rectangular waveguides is presented. The slot is longitudinal and offset from the center line in the main guide and is centered transverse in the branch guide. Pertinent integral equations are developed, taking into account finite wall thickness. The integral equations are then solved for the aperture electric field. Coupling slot characteristics are obtained, including the resonant length and dominant mode scattering. Numerical results for resonant length and scattering parameters are presented over a range of offsets, waveguide dimensions, and frequencies.

I. INTRODUCTION

A PRINCIPAL application of slot couplers involves the transfer of power from a main waveguide to a family of crossed branch waveguides that contain the radiating slots of a planar array. Watson [1] was the first to study such waveguide slot couplers. He derived approximate expressions for several equivalent circuit representations. Vu Khac and Carson [2] later presented a rigorous analysis of a slot coupler using an integral equation. They solved the integral equation by the method of moments and obtained numerical results for longitudinally offset and transverse slot couplers. Harrington and Mautz presented a general formulation for aperture problems in terms of the method of moments [3]. It applies to any two regions isolated except for coupling through the aperture.

A commonly employed coupling slot (longitudinal/transverse) for planar slot array application is longitudinal and offset from the broad wall center line in the main waveguide and centered transverse in the branch waveguide. Power coupled to the branch waveguide is controlled by the amount of offset. Adjacent coupling slots in the main waveguide are usually one half guide wavelength apart and their offsets alternate on each side of the main waveguide center line. This arrangement serves to excite all the branch waveguides in phase. The longitudinal/transverse coupler behaves very nearly as a shunt element in the

main waveguide and as a series element in the branch waveguide, and so it is generally known as a *shunt-series* coupling slot.

Park *et al.* [4] investigated the longitudinal/transverse coupling slot by solving the pertinent integral equation for the aperture electric field using the method of moments. They employed a pulse-expansion point-matching approach, dividing the slot into a number of rectangular subdomains. Results on resonant length and resonant conductance in the main waveguide were presented. Senior [5] studied the problem of higher order mode coupling between a longitudinal/transverse coupling slot and a pair of straddling radiating slots in the branch waveguide, using an integral equation formulation. Piecewise-sinusoidal Galerkin and global Galerkin methods for solving the integral equations were employed by Senior. Wall thickness effects were ignored in both of the above analyses [4], [5]. Recently, a centered-inclined coupling slot was analyzed rigorously [6]. There remains the need to investigate the characteristics of a longitudinal/transverse coupling slot and this paper addresses that problem. Results to be presented provide a better understanding of the properties of this useful coupler in planar array applications. It may also be used as an unequal power divider in microwave circuitry applications [4].

II. ANALYSIS

Fig. 1 shows two rectangular waveguides, a branch line guide designated by ports 3 and 4, and a main line guide (ports 1 and 2) situated underneath and perpendicular to the branch waveguide. A coupling slot is cut in the common broad wall between the two waveguides. The slot is offset by an amount δ from the center line of the main waveguide, and it is centered transverse in the branch waveguide. The slot is of length $2l$ and width w . Both waveguides are assumed to be composed of perfectly conducting walls and to be filled with homogeneous isotropic lossless dielectrics whose constitutive parameters are μ_0 , ϵ_m and μ_0 , ϵ_b in the main and branch lines respectively. The common broad wall thickness t between the lower and upper slot apertures will be referred to as the wall thickness in further discussions.

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The author is with the Department of Electrical and Computer Engineering, California State University, Northridge, Northridge, CA 91330. IEEE Log Number 8929329.

representative of the $2N$ testing functions is

$$w_p(\xi, \zeta) = \delta(\xi) \sin[p\pi(\zeta + l)/(2l)] \quad (6)$$

where p takes values from 1 through N for each aperture. The matrix set is expressed in terms of partitioned matrices:

$$\begin{bmatrix} [Y_{11}] & [Y_{12}] \\ [Y_{21}] & [Y_{22}] \end{bmatrix} \begin{bmatrix} [A] \\ [B] \end{bmatrix} = \begin{bmatrix} [I] \\ [0] \end{bmatrix}. \quad (7)$$

In the above $[Y_{11}] = [Y^{\text{int}}] - [Y_s^c]$. A typical element in $[Y^{\text{int}}]$ is

$$Y_{pq}^{\text{int}} = - \int_{-l}^l \sin[p\pi(\zeta + l)/(2l)] \cdot \iint_{S_1} G_{zz} \sin[q\pi(\zeta' + l)/(2l)] d\zeta' d\xi' d\zeta \quad (8)$$

and $[Y_s^c]$ is a diagonal matrix [8]:

$$[Y_{22}] = [Y^{\text{ext}}] + [Y_s].$$

A typical element in $[Y^{\text{ext}}]$ is

$$Y_{pq}^{\text{ext}} = \int_{-l}^l \sin[p\pi(\zeta + l)/(2l)] \cdot \iint_{S_2} G_{xx} \sin[q\pi(\zeta' + l)/(2l)] d\zeta' d\xi' d\zeta. \quad (9)$$

$[Y_{12}]$ and $[Y_{21}]$ are also diagonal matrices [8]. The integrals in (8) and (9) are evaluated analytically. The resulting expressions for Y_{pq}^{int} and Y_{pq}^{ext} contain double summations in terms of waveguide modes, propagating and evanescent. For computational purposes, the slowly converging series have been expressed in terms of more rapidly converging series, based on their asymptotic behavior [11].

From the solution of the matrix equations, the aperture electric field is determined. Dominant mode scattering in both waveguides is then obtained. Computed results in this work assume both guides to be air filled with identical cross sections except for the data in Table IV. However, the analysis is more general, and is valid for dissimilar waveguides, dissimilarly filled.

III. DISCUSSION AND NUMERICAL RESULTS

A. Resonant Length

The importance of determining the resonant length of a slot to high precision has been explained by Stern and Elliott [12]. The longitudinal/transverse coupling slot behaves nearly like a shunt element in the main waveguide. The shunt representation becomes poorer for smaller waveguide b dimensions and larger offsets. This behavior has been observed previously for a longitudinal radiating slot on the broad wall of a rectangular waveguide [12]. For compound radiating and coupling slots, resonance has been defined by the condition wherein the forward-scattered TE_{10} mode is out of phase with the incident TE_{10} mode [8], [9]. This condition corresponds to a maximum of energy radiated or coupled into the branch waveguide, and it applies to shunt and series elements also [1]. Use of this

TABLE I
PHASE OF S_{11} AT RESONANCE

OFFSET (Inch)	S_{11} phase in degrees		
	$b=0.4"$	$0.2"$	$0.1"$
0.05	178.9	177	172
0.10	178.7	176.3	167.6
0.15	178.5	174.6	--
0.20	178.1	172.3	--
0.25	177.7	167.8	--

$a = 0.9$ in., $w = 0.0625$ in., $t = 0.03$ in., $f = 9.3$ GHz.

definition of resonance for the longitudinal/transverse coupling slot is consistent with compound coupling slot results for the limiting case of zero tilt [9]. A second definition of resonance is based on the condition wherein the backscattered TE_{10} mode is out of phase with the incident TE_{10} mode. This definition has been employed in the literature for shunt-type elements, longitudinal/transverse slot couplers, and longitudinal radiating slots [4], [5], [12].

Resonant length based on the forward-scattered wave phase is difficult to determine experimentally whereas that based on the backscattered wave phase is easy to measure. For an ideal shunt element both definitions are the same. Since longitudinal radiating slots and coupling slots do not behave as perfect shunt elements, the two definitions result in different resonant lengths. The resonance definition based on forward-scattered wave phase predicts slightly longer slots.

Table I shows the phase of the backscattered wave at resonance when the forward-scattered wave is out of phase with the incident TE_{10} mode wave. This definition of resonance is used throughout this paper except for comparison of theory and experiment, and for comparing the two resonance conditions. For a longitudinal/transverse resonant coupling slot in a standard X-band waveguide, the phase of the backscattered wave is very close to 180° . For smaller b dimensions and greater offsets, the phase of the backscattered wave deviates substantially from the phase of the forward-scattered wave. For such slots, resonant lengths based on the two definitions differ significantly. Maximum power coupling occurs for a slot length between the two resonant lengths.

Table II shows a comparison of the two resonance conditions. The forward-scattered TE_{10} mode is labeled $S_{21} - 1$. For the resonant condition based on forward scattering, $S_{21} - 1$ is not given in Table II since $|S_{21} - 1| \approx |S_{11}|$ and its phase is 180° . Similarly, for resonance based on backscattering, S_{11} is not given. The phase of the aperture electric field at the center of the slot, ϕ , and phase variation of the electric field along the slot, $\Delta\phi$, are also shown. It can be seen that ϕ varies significantly from 90° for

TABLE II
A COMPARISON OF TWO RESONANCE CONDITIONS

δ	forward scattering definition			backscattering definition		
	$2k_0 l_{\text{res}}$	S_{11}	$\phi, \Delta\phi$	$2k_0 l_{\text{res}}$	S_{21}^{-1}	$\phi, \Delta\phi$
0.05"	3.367	0.0301 172.9°	86.0°, 1.8°	3.307	0.0301 -173.5°	93.3°, -1.3°
0.10"	3.682	0.108 168.6°	84.2°, 3.9°	3.505	0.108 -168.7°	94.5°, -2.9°
0.15"	-	-	-	3.843	0.208 -163.4°	97.5°, -8.5°

$a = 0.895$ in., $b = 0.13$ in., $w = 0.06$ in., $t = 0$, $f = 9.3$ GHz.

TABLE III
A NONRESONATING CONDITION
(FORWARD-SCATTERED WAVE DEFINITION)

$2k_0 l$	S_{11}	S_{21}^{-1}
3.994	0.205 174.0°	0.208 -168.3°
4.077	0.207 170.6°	0.208 -170.1°
4.243	0.210 165.9°	0.207 -171.4°
4.326	0.211 163.8°	0.206 -171.9°

$a = 0.895$ in., $b = 0.13$ in., $w = 0.06$ in., $t = 0$, $f = 9.3$ GHz, $\delta = 0.15$ in.

small b dimensions and for large values of δ . The phase of the aperture electric field even at resonance is found to be nonuniform along the slot. The phase varies rapidly in the region $z < 0$ and somewhat slowly in the region $z > 0$. A similar behavior was observed for longitudinal radiating slots by Stern and Elliott [12]. For $\delta > 0.15$ in., the slot does not resonate, as per the forward-scattering definition, even when the length equals the waveguide a dimension. This is a limiting value in the branch waveguide. This condition is illustrated by the data in Table III. The forward-scattered wave phase varies very slowly and does not reach the value of 180°. The backscattered wave also exhibits such a behavior for larger values of offset.

The longitudinal/transverse slot coupler having a large offset in a reduced-height guide can be operated at any one of the above-mentioned resonant frequencies. The aperture electric field distribution of such a slot deviates substantially from a half cosine shape; hence the global sinusoidal Galerkin technique may not be accurate. A piecewise-sinusoidal Galerkin method may be a better alternative [5]. Entire domain vector expansion functions previously used for radiating slots [13] would provide accurate results for slots having nonzero wall thickness.

Fig. 3 shows the normalized resonant length, $2k_0 l_{\text{res}}$, for a longitudinal/transverse slot coupler as a function of slot offset for different waveguide b dimensions and wall thicknesses. Here k_0 is the free-space wavenumber and $2l_{\text{res}}$ is the physical length of the slot. For all cases, resonant length increases with offset. This variation is more rapid in reduced-height waveguides. For a quarter-height X-band

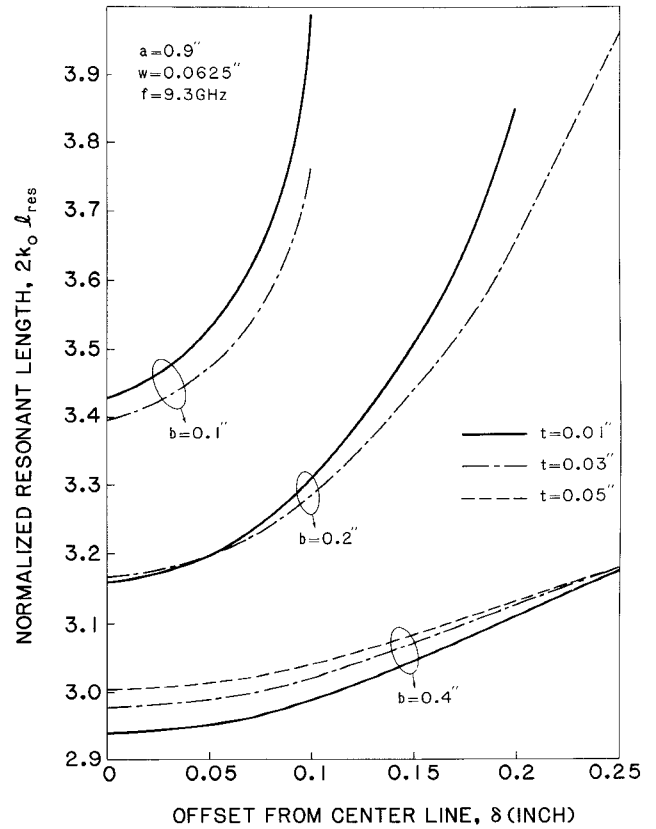


Fig. 3. Normalized resonant length as a function of offset.

waveguide, the coupling slot does not exhibit a resonant behavior for $\delta \geq 0.150$ in. However, with the backscattering type of resonance definition, it is found that the slot exhibits resonance for a larger range of δ . Wall thickness has the effect of reducing the resonant length if $2l > \lambda/2$, and it has the opposite effect if $2l < \lambda/2$. Here λ is the free-space wavelength. Similar wall thickness effects were found previously for broad wall resonant slots, both radiating and coupling types [6], [8]–[10]. The resonant lengths of longitudinal/transverse coupling slots are significantly longer than centered-inclined coupling slots, especially for smaller b dimensions [6].

Normalized resonant length decreases with frequency as shown in Fig. 4. The reduction of $2k_0 l_{\text{res}}$ with frequency is significantly greater for slot couplers in reduced-height waveguides. The effect of slot width on resonant length is illustrated by Fig. 5. In a reduced-height waveguide, a wider slot is seen to be of longer resonant length. Slots in a standard-height waveguide exhibit a similar behavior for the resonant length dependence on slot width. However, when the offset is small, a wider slot in a standard height waveguide is seen to be of shorter resonant length.

B. Dominant-Mode Scattering

The backscattered wave amplitude at resonance, $|S_{11}|$, is primarily dependent on the slot offset and is practically insensitive to the waveguide b dimension and to wall thickness. Fig. 6 shows the variation of $|S_{11}|$ with slot offset of a standard X-band waveguide and for a half-height guide. The forward-scattered wave amplitude at

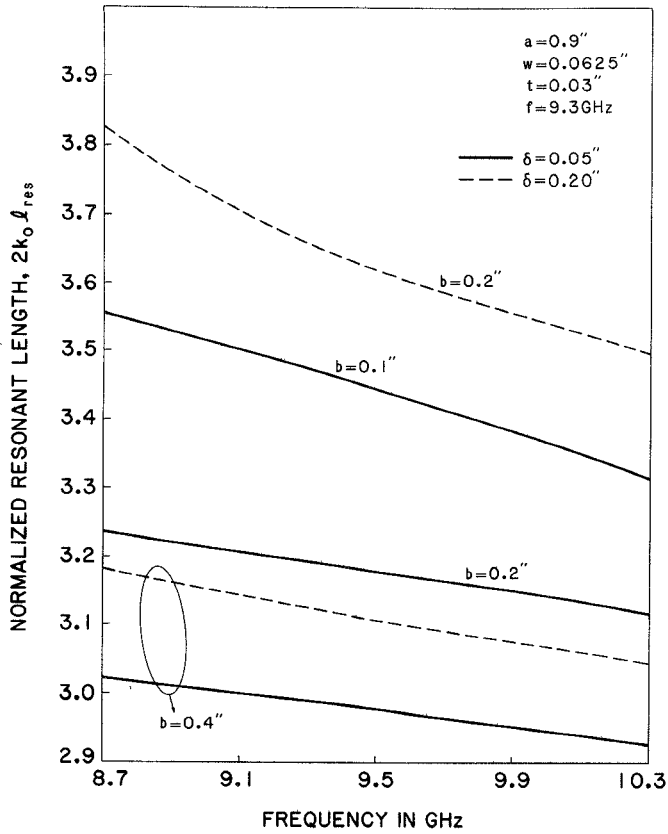


Fig. 4. Normalized resonant length as a function of frequency.

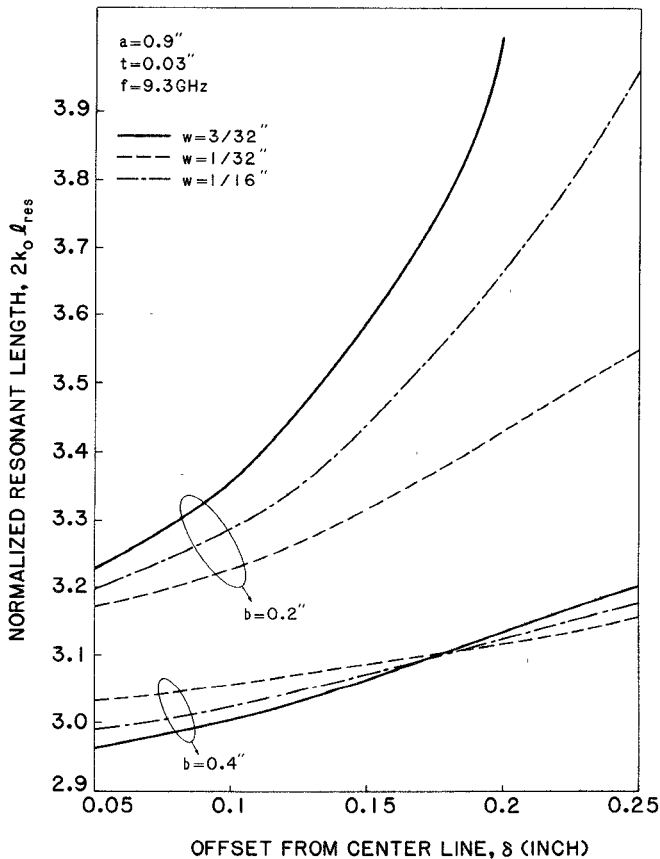
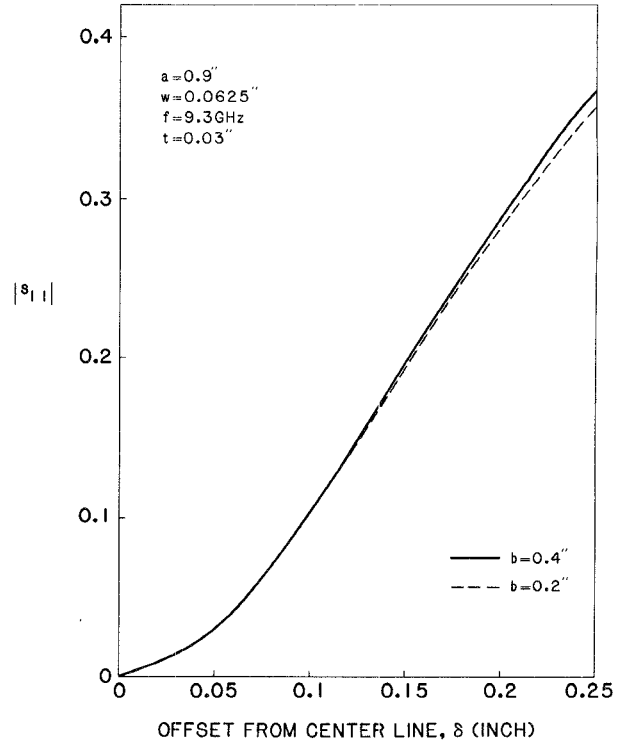


Fig. 5. Normalized resonant length for different slot widths.

Fig. 6. $|S_{11}|$ at resonance as a function of offset.

resonance, $|S_{21} - 1| \approx |S_{11}|$. Some examples of the phase of S_{11} at resonance are given in Table I. From that table one can see that the shunt representation in the main waveguide is very good for coupling slots in a standard-height waveguide with small offsets. For larger offsets and smaller waveguide b dimensions, the shunt representation is found to be poor. In the branch waveguide, the slot representation as a series element is found to be excellent. The scattered wave amplitude in each branch waveguide at resonance is obtained using a power balance, that is, $|S_{31}| = |S_{41}| = [|S_{11}| \{1 - |S_{11}|\}]^{1/2}$. At resonance, if the phase of S_{11} is ψ , the phases of S_{31} and S_{41} are approximately $\psi/2$ and $\pi + \psi/2$ respectively. When slot offset becomes negative, S_{31} and S_{41} have phases that are $\pi + \psi/2$ and $\psi/2$ respectively. Fig. 7 shows $|S_{11}|$ at resonance for a longitudinal/transverse coupling slot as a function of frequency for different offsets. Over a 17 percent frequency range, $|S_{11}|$ at resonance is seen to drop by nearly 50 percent. For a centered-inclined coupling slot, $|S_{11}|$ at resonance drops by no more than 22 percent for a similar frequency range [6]. This shows that at higher frequencies longitudinal/transverse slot couplers require a greater offset, and hence a longer resonant length.

C. Off-Resonant Characteristics

Over a small frequency range around resonance, the phase of the dominant-mode scattered waves in the main line and branch line and the phase of the aperture electric field track each other. Thus from a knowledge of the phase variation of S_{11} around resonance the characteristics of S_{31} , S_{41} and forward-scattered wave in the main line can be deduced. Fig. 8 shows the phase variation of S_{11} over a 1 percent frequency change from resonance. Thin wall slots are seen to exhibit a smaller phase variation around

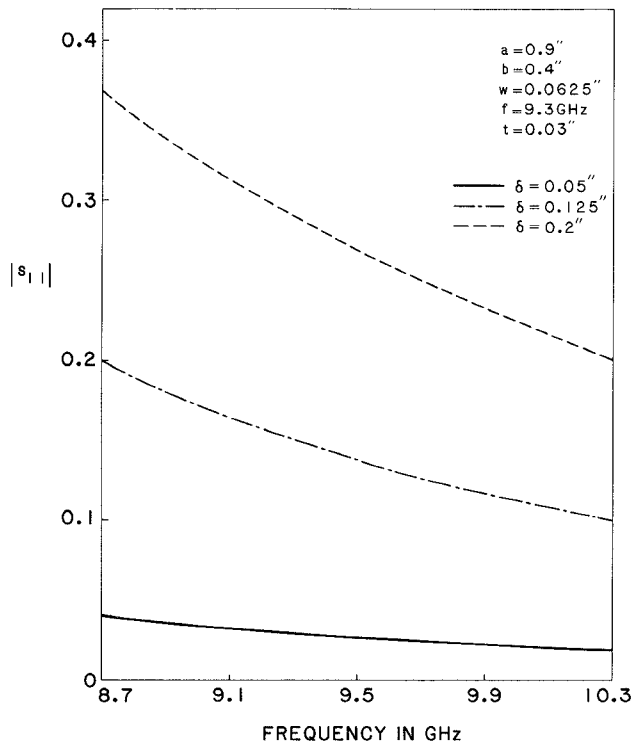


Fig. 7. $|S_{11}|$ at resonance as a function of frequency.

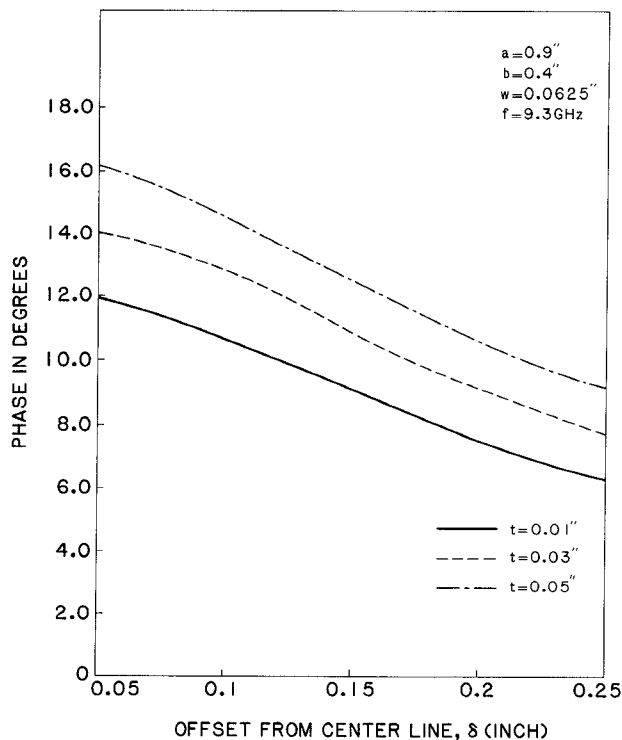


Fig. 8. Phase variation of S_{11} over a 1 percent frequency change from resonance.

resonance. Phase variation for a slot of larger offset is less than that for a slot of smaller offset. This differential phase introduces phase errors in the branch line excitation in a planar slot array, and causes degradation in pattern performance off resonance. Centered-inclined slot couplers introduce fewer phase errors off resonance, and hence have better performance [6].

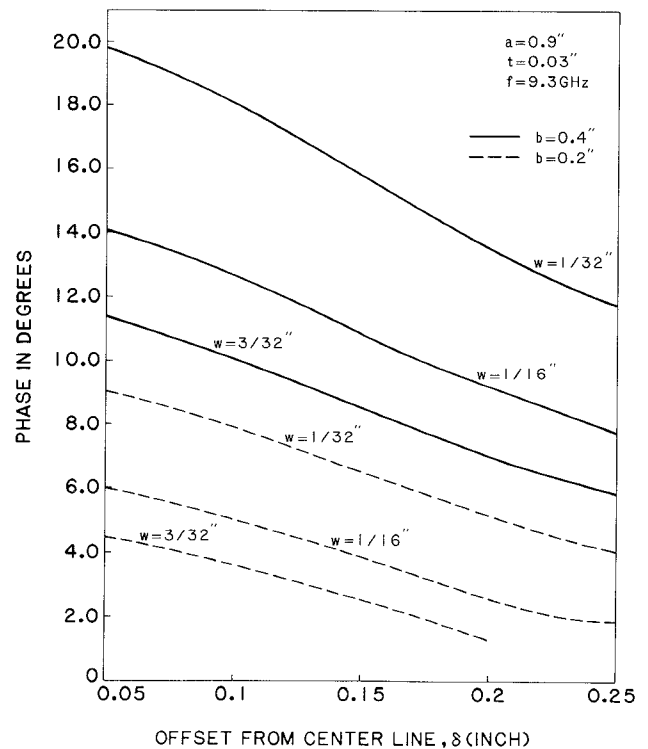


Fig. 9. Phase variation of S_{11} over a 1 percent frequency change for different slot widths.

Fig. 9 shows the phase variation of S_{11} over a 1 percent frequency change around resonance for coupling slots of different slot widths. Wider slots possess the desirable characteristic of smaller phase variation off resonance. Here again the longitudinal/transverse slot couplers have larger differential phase than centered-inclined slot couplers [6].

D. Validation of the Theoretical Model

All the results discussed in this paper were obtained with ten expansion modes in the global Galerkin technique. The effect of higher order Galerkin modes beyond ten is found to be insignificant. Josefsson has reported a similar finding for longitudinal radiating slots [10]. Computed values of resonant length and resonant conductance obtained from this theory were in good agreement with previously reported values of resonant length [4] and corrected values of resonant conductance [4], [14]. In addition, results obtained from this theory were checked against those of a compound coupling slot for $\theta = 0$ [9]. Table IV shows a comparison of results computed from this theory and those obtained experimentally. The experimental work was performed by Park *et al.* at Hughes Aircraft Company, Canoga Park, CA [14]. Since resonance was established experimentally from the condition that the phase of S_{11} be 180° , the computed resonant parameters were based on the same definition. Excellent agreement between computed $|S_{11}|$ and experimentally measured $|S_{11}|$ is found. A discrepancy occurs for $\delta = 0.09$ in., which may be due partly to experimental error and tolerances and partly to minor imperfections in theory. Computed values of reso-

TABLE IV
COMPARISON OF THEORY AND EXPERIMENT

Offset, δ	Theory		Experiment	
	$ S_{11} $	$2l_{res}$	$ S_{11} $	$2l_{res}$
0.09"	0.087	0.653"	0.083	0.645"
0.18"	0.25	0.717"	0.251	0.722"
0.27"	0.387	0.804"	0.383	0.835"

Main line waveguide: 0.9 in. \times 0.2 in.

Branch line waveguide: 0.881 in. \times 0.2 in.

$w = 0.06$ in., $t = 0.01$ in., $f = 9.3$ GHz.

nant length show very good agreement with experimental results except for the large offset ($\delta = 0.27$ in.) case. For large offsets and small b dimensions, the aperture electric field deviates substantially from a half cosinusoidal distribution. The inability to capture such a distribution by the sinusoidal global Galerkin technique may perhaps explain the discrepancy in resonant length computed for $\delta = 0.27$ in. A piecewise-sinusoidal Galerkin would be a better approach for large offsets. Large offsets are seldom employed in slot arrays; hence the global Galerkin method is well suited for most practical applications.

IV. CONCLUSIONS

This paper has presented a rigorous analysis of a longitudinal/transverse slot coupler. Integral equations for the aperture electric field have been developed, taking into account finite wall thickness. Numerical results for resonant length, dominant-mode scattering, and phase variation around resonance have been presented over a range of values of the waveguide b dimension, wall thickness, slot offset, and frequency. These results have applications to waveguide-fed slot arrays and microwave circuitry.

A series representation of this coupling slot in the branch waveguide is found to be excellent whereas shunt representation in the main guide is inaccurate for small waveguide b dimensions and large offsets. In a planar array, the coupling slot can be operated at a resonant frequency based on the forward-scattered wave or the backscattered wave type definition. Scattering characteristics of the slot should be properly taken into account in a design procedure.

For slot array applications, the longitudinal/transverse coupling slots are found to have characteristics inferior to those of centered-inclined slot couplers [6]. In this paper, the scattering characteristics of the former were obtained based on the forward-scattered wave resonance definition. The above conclusion should hold even if the resonant characteristics are computed by employing the backscattered wave definition.

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Sembiam R. Rengarajan (S'77-M'80-SM'85) received the Ph.D. degree in electrical engineering from the University of New Brunswick, Fredericton, NB, Canada, in 1980.

He was an antenna development engineer at Bharat Electronics Ltd., Ghaziabad, U.P., India, during the years 1971-1976. Since 1980, he has been with the California State University, Northridge, presently serving as Professor in the Department of Electrical and Computer Engineering. From June 1983 to August 1984, he was with the Spacecraft Antenna Research Group of the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA. During 1987-1988, he was a Visiting Professor at the University of California, Los Angeles, on sabbatical leave. He has been a consultant to Hughes Aircraft Company and NASA/Jet Propulsion Laboratory, and a visiting researcher at UCLA. He is listed in the 21st edition of *Marquis Who's Who in the West*. His research interests include applied electromagnetics and numerical techniques.

Dr. Rengarajan has served the Los Angeles chapter of the IEEE Antennas and Propagation Society as Chairman (1983-1984), Vice Chairman (1982-1983), and Secretary/Treasurer (1981-1982). His professional memberships include URSI Commission B, the American Society of Engineering Education, and Sigma Xi.